

Update on nuclear structure effects in light muonic atoms

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Abstract We present calculations of the nuclear structure corrections to the Lamb shift in light muonic atoms, using state-of-the-art nuclear potentials. We outline updated results on finite nucleon size contributions.

Keywords Proton radius puzzle · Muonic atoms · Nuclear polarizability

1 Introduction

The root-mean-square (RMS) charge radius of the proton r_p has been extracted to unprecedented precision from recent measurements of the Lamb shift (2S-2P) transition in muonic hydrogen μH . The extracted r_p value is smaller by 7σ with respect to the CODATA value, which is based on $e\text{H}$ spectroscopy and $e\text{-}p$ scattering [1,2]. This unresolved discrepancy between the electron-based and the muon-based values is known as “the proton radius puzzle”, and challenges our understanding of physics based on the standard model. The CREMA collaboration at PSI has undertaken the task of extracting the nuclear RMS charge radius R_c from measurements of the Lamb shift in several hydrogen-like muonic systems, with the aim to improve the accuracy in the determination of R_c . These new measurements may shed light on the puzzle by studying whether the discrepancy between the nuclear charge radii measured using electrons and muons persists or varies as a function of the atomic mass A and charge number Z . For $A \geq 2$ these measurements are affected by nuclear structure corrections. Ultimately, these corrections determine the attainable precision of R_c , and it is therefore important to obtain the most accurate nuclear structure corrections with reliable uncertainties.

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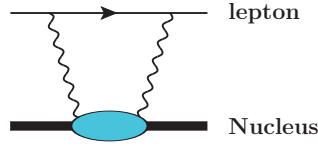


Fig. 1 The muon-nucleus two-photon-exchange. The bulb denotes the excitation of the nucleus in the intermediate states between the two photons.

The Lamb shift is related to R_c by

$$\Delta E = \delta_{\text{QED}} + \delta_{\text{FS}}(R_c) + \delta_{\text{TPE}}. \quad (1)$$

The three terms, given in order of decreasing magnitude, are: the QED contributions from vacuum polarization, lepton self energy, and relativistic recoil; the leading correction due to the finite size of the nucleus, $\delta_{\text{FS}}(R_c) = \frac{m_r^3(Z\alpha)^4}{12} R_c^2$ (in $\hbar = c = 1$ units), from one-photon-exchange; and the two-photon exchange (TPE) contribution depicted in Fig. 1. The contributions from this diagram are divided into the elastic Zemach term and the inelastic polarization term, *i.e.*, $\delta_{\text{TPE}} = \delta_{\text{Zem}} + \delta_{\text{pol}}$, which can be further separated into nuclear (δ^A) and nucleonic (δ^N) components, *i.e.*, $\delta_{\text{TPE}} = \delta_{\text{Zem}}^A + \delta_{\text{pol}}^A + \delta_{\text{Zem}}^N + \delta_{\text{pol}}^N$. In Refs. [3,4,5,6] we have provided values for the various terms in δ_{TPE} for the hydrogen-like muonic systems with $2 \leq A \leq 4$, based on *ab initio* calculations with various state-of-the-art models for the nuclear Hamiltonian and newly developed methods [7]. This has enabled us to provide improved precision and uncertainty estimates compared to previous results. Here we provide details on recent recalculations of δ_{Zem}^N and the corrections to δ_{pol}^A due to the finite size of the nucleons.

2 Nucleon size corrections to the nuclear polarizability

The calculation of δ_{pol}^A is portrayed in Refs. [3,4], where the various terms contributing to δ_{pol}^A are obtained in a multipole expansion formalism

$$\delta_{\text{pol}}^A = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + \delta_{NS}^{(1)} + \delta_{NS}^{(2)}. \quad (2)$$

The terms $\delta^{(0)}, \delta^{(1)}, \delta^{(2)}$ denote the leading, sub-leading, and 2nd-order corrections, respectively, all calculated in the point-nucleon limit. The terms $\delta_{NS}^{(1)}$ and $\delta_{NS}^{(2)}$ denote the respective nucleon size (NS) corrections. The details of the calculation and formula for each of these terms are illustrated in Ref. [3]. Some of the terms that contribute to the NS corrections have been updated in Ref. [6]; below we provide more details as well as further updates.

When the finite sizes of the nucleons are considered, the Coulomb interaction with the point-protons is substituted by an integration over the internal charge distributions of the nucleons. In Refs. [3,4,5] the NS corrections were derived in momentum-space, using the low Q^2 approximation for the nucleon

form factors. They were re-derived in Ref. [6] in coordinate-space, as was done by Friar in Ref. [8]. The results of this treatment are identical to those of Refs. [3, 4, 5], with the exception of a term that vanishes in the momentum-space treatment used there. This term is related to the 3rd Zemach moments of the nucleons. The contribution from the neutrons is negligible, and the contribution from the protons is estimated from the corresponding term in muonic hydrogen $\delta_{Zem}(\mu H)$ according to

$$\delta_{Zem}^N(\mu A) = \left(\frac{Zm_r(\mu A)}{m_r(\mu H)} \right)^4 \times \delta_{Zem}(\mu H). \quad (3)$$

The scaling factor in the above equation can be extracted from Ref. [8]. However, it differs from the one used, e.g., in Ref. [9] for both the elastic and inelastic nucleon TPE, and calls for further investigation.

Next we detail the 2nd order NS corrections $\delta_{NS}^{(2)}$. The neutron-neutron contribution is negligible, therefore only proton-proton $\delta_{pp}^{(2)}$ and neutron-proton $\delta_{np}^{(2)}$ terms contribute. The former is expressed, as in Ref. [3], by

$$\delta_{pp}^{(2)} = -\frac{8\pi}{27} m_r^5 (Z\alpha)^5 r_p^2 \int_{\omega_{th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} S_{D_1}(\omega), \quad (4)$$

using the dipole response

$$S_{D_1}(\omega) = \sum_{f \neq 0} |\langle f | \hat{D}_1 | 0 \rangle|^2 \delta(E_f - E_i - \omega), \quad (5)$$

where $\hat{D}_1 = \frac{1}{Z} \sum_a r_a Y_1(\hat{r}_a)^{\frac{1+\tau_a^3}{2}}$, with Y_1 the rank-1 spherical harmonic. The latter, however, takes the form

$$\begin{aligned} \delta_{np}^{(2)} &= -\frac{8\pi}{27} m_r^5 (Z\alpha)^5 \frac{N}{Z} r_n^2 \int_{\omega_{th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} S_{D_p D_n}(\omega) \\ &= \frac{8\pi}{27} m_r^5 (Z\alpha)^5 r_n^2 \int_{\omega_{th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} S_{D_1}(\omega), \end{aligned} \quad (6)$$

where

$$S_{D_p D_n}(\omega) = \sum_{f \neq 0} \langle 0 | \hat{D}_n^\dagger | f \rangle \langle f | \hat{D}_p | 0 \rangle \delta(E_f - E_i - \omega) \quad (7)$$

contains the protonic dipole operator $\hat{D}_p = \hat{D}_1$ and the neutronic dipole operator $\hat{D}_n = \frac{1}{N} \sum_a r_a Y_1(\hat{r}_a)^{\frac{1-\tau_a^3}{2}}$. The isoscalar part of both operators is proportional to the nuclear center of mass motion, and vanishes between the nuclear ground and excited states. Thus, we obtain $S_{D_p D_n}(\omega) = -(Z/N) S_{D_1}(\omega)$, which was used in the last step of Eq. (6). The expression in Eq. (6) supersedes the corresponding ones in [3, 6]. The results in Eqs. (4) and (6) coincide with the corresponding expression for the special case of deuterium given in Ref. [10], and were used in a recent summary of the theory related to the μD experiment [9].

3 Results

Integrating the new δ_{Zem}^N and $\delta_{NS}^{(2)}$ discussed above into the results of Refs. [3, 4, 6] we obtain updated values for μD , $\mu^4 He^+$, $\mu^3 He^+$, and $\mu^3 H^+$. In Table 1 below we present updated results for the individual terms in δ_{TPE} for all $2 \leq A \leq 4$ hydrogen-like muonic systems, where the results for $A = 2, 4$ are taken from Refs. [3, 4, 11], and for $A = 3$ the results given here are taken from Ref. [6] and supersede it due to Eq. (6) and a correct implementation of Eq. (3). More details will be given in Ref. [12]. For completeness we include δ_{pol}^N in Table 1, where for μD its value and uncertainty were extracted from data in Ref. [13], and for other systems they were estimated based on their values in μH , as explained in Ref. [6].

Table 1 Contributions to δ_{TPE} of the Lamb shift in light muonic atoms, in meV. The uncertainty associated with each value is given in brackets and includes the numerical, nuclear model, and atomic physics uncertainties. Due to cancellation of the Zemach term, the uncertainty in δ_{TPE} can differ from the quadrature sum of the corresponding terms in the table. A detailed analysis of the uncertainties is given in Ref. [6].

	δ_{Zem}^A	δ_{pol}^A	δ_{Zem}^N	δ_{pol}^N	δ_{TPE}
μD	-0.424(3)	-1.245(19)	-0.030(2)	-0.028(2)	-1.727(20)
$\mu^3 H$	-0.227(6)	-0.473(17)	-0.033(2)	-0.034(16)	-0.767(25)
$\mu^3 He^+$	-10.49(24)	-4.17(17)	-0.52(3)	-0.28(12)	-15.46(39)
$\mu^4 He^+$	-6.29(28)	-2.36(14)	-0.54(3)	-0.38(22)	-9.58(38)

The results in Table 1 provide rigorous estimates of the nuclear structure corrections to the Lamb shift in the lightest hydrogen-like muonic systems, and a careful treatment of their associated uncertainties. Together with the highly precise determinations of the other terms in Eq. (1), these results will allow the nuclear charge radii of these isotopes to be determined with significantly improved precision. This development should be insightful not only towards the resolution of the proton radius puzzle but also to probe possible effects due to the neutrons.

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